A Stereoscopic Representation of Impossible Rectangle Twisted Torus Figure

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1. Introduction

Impossible figure is a figure which looks realizable at first glance, but is not actually realizable as perceived by the human eyes. Optical illusion object is a three dimensional object made to be an impossible figure when viewed from a specific viewpoint. The viewpoint that realizes an optical illusion is called as an illusion viewpoint. An optical illusion object is not perceived as an impossible figure when a viewpoint moves away from an illusion viewpoint.

In this paper, we propose a shape modeling method and an animating method for an optical illusion object that shapes an impossible rectangle twisted torus figure, which is a kind of impossible figures, according to the illusion viewpoint. The proposed method realizes an interactive and stereoscopic representation of the impossible figure, which has not been dealt with in previous research, moved in a wide range of illusion viewpoints.

2. 3D Shape Model for Optical Illusion Object

Three dimensional shape model of twisted torus, which is one of the optical illusion objects, consists of multiple quadrangular prisms (prism in short) as shown in Fig.1. Shape model fundamentally is a ring shape. This has a shape with a ring with area cut apart to model an impossible object as an existent three-dimensional object. We assign numbers to prisms, and the starting point of the number is the area cut part. In the example of Fig.1 (a), the shape model consists of four prisms in total. We assign number to each prism: 1 to 4.

Two prisms fundamentally connecting with each other are called 'start prism' and 'end prism' respectively. In the example of Fig.1, prism 1 and 4 are the start prism and the end prism respectively. There is a gap between prism 1 and 4, and they are not connected (Fig.1 (b)).

The plane consisting of the first two prisms is called 'front face'. In Fig.1(a), the front face consists of prism 1 and 2. From an illusion viewpoint, the prisms behind the front face is called 'flexible quadrangular prism' (*flexible prism* in short). In Fig.1(a), prism 3 and 4 are the flexible prisms. We transform the shape model into optical illusion object by stretching the flexible prism 3 and 4 according to the given viewpoint.

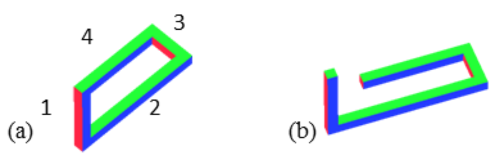

Fig1. Optical Illusion Objects

3. Illusion Viewpoint and Projection

The sphere containing an optical illusion object is called 'bounding sphere'. The illusion viewpoint moves on the upper half of the bounding sphere except the pole. The illusion viewpoint looks at the center of the sphere all the time. The illusion viewpoint is given by a latitude ϕ, and longitude θ on the bounding hemi-sphere. The illusion viewpoint is expressed as (ϕ, θ), where $0 \leq \phi < \frac{\pi}{2}, 0 \leq \theta < 2\pi$.

The parallel projection is used for the shape model seen from the illusion viewpoint by 3DCG.

4. Modeling of Optical Illusion

We stretch the flexible prism of a shape model in the depth and the horizontal direction so a shape model can be seen as an optical illusion object when the model is seen from an arbitrary illusion viewpoint (ϕ, θ). In this chapter, we explain the method to transform a shape model concretely so it can be seen as an optical illusion object using Fig.2. As shown in Fig.2(a), the shape model of an optical illusion object in Fig.1 is projected on X-Z plane.

The illusion viewpoint with the longitude of θ, that is, the viewpoint $(\phi, 0)$ is called 'initial illusion viewpoint'. We regard that the shape model is seen as an optical illusion object when it is seen from the initial illusion point$(\phi, 0)$. Here, when the shape model is projected parallel toward the illusion viewpoint, the vertex $F(\phi, 0)$ of starting prism and the vertex $G(\phi, 0)$ of ending prism become the same value in the projection plane, and both vertexes look connected.

The way for the shape model to be seen as an optical illusion object, when the viewpoint moves to (ϕ, θ), is to transform the shape model to have vertex $F(\phi, \theta)$ and $G(\phi, \theta)$ from a viewpoint (ϕ, θ) with the same value. So, as in Fig.2 (b), the ending prism 4 is moved to the prism 4' in the depth direction parallel to X axis and Z axis, and stretched in the X axis direction as the way to realize the position of the vertex G as below:

$$G_X(\theta) = r \sin\theta + F_X(0) \qquad (1)$$

$$G_Z(\theta) = -r \cos\theta + F_z(0) \qquad (2)$$

Next, an optical illusion object is complete by stretching the prism 3 connected to the ending prism in the depth (Z axis) direction.

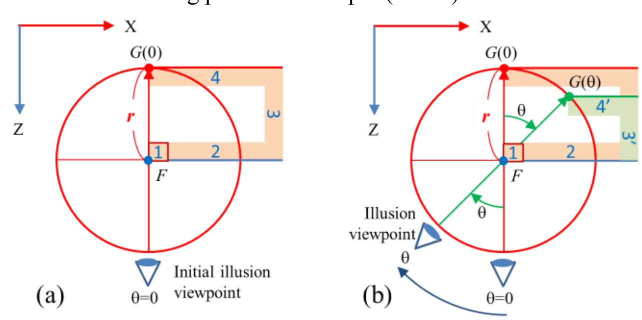

Fig2. $G(\theta)$ Coordinate Change Viewed from X-Y Plane

5. Stereoscopic Vision of Impossible Rectangle Twisted Torus Figure

With the method mentioned above, it is possible to model an impossible rectangle twisted torus figure. In this chapter, we realize the stereoscopic vision of an impossible figure by using the binocular disparity as a difference in longitude of the illusion viewpoint.

The stereoscopic vision of an impossible figure is realized by modeling optical illusion objects with different longitude of illusion viewpoint respectively for the right eye and the left eye to make stereoscopic representation of the results from each illusion viewpoint. Fig.3 is a stereoscopic image generated by our method. Further, it is possible to move illusion viewpoint interactively, and the interactive stereoscopic view is also possible. See the attached movie.

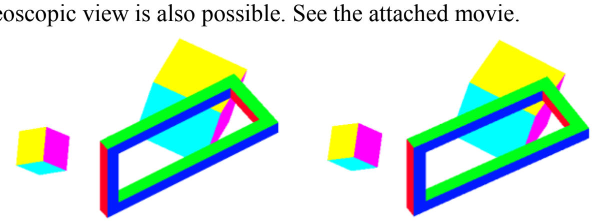

Fig3. Generated Stereoscopic Image (For the Parallel Method)

6. Conclusion

We realized the stereoscopic representation of the interactive animation of an impossible twisted torus figure. We have implemented an interactive stereoscopic animation system of an impossible figure on Nintendo 3DS game system.

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